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RELIABILITY METHOD COUPLED WITH FINITE ELEMENT ANALYSIS FOR STRUCTURAL PROBLEMS WITH IMPLICIT RESPONSE FUNCTIONS

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Abstract

This paper deals with a combination between reliability program and finite element analysis software used to evaluate statistical characteristic of the response of the mechanical structure. The probabilistic transformation method (PTM) is an efficient reliability method to solve problems of mechanical systems with uncertain parameters. This method is readily applicable in the case, where the expression between input and output of structures are available in explicit analytical form. However, the situation is much more involved, when it is necessary to perform the evaluation of implicit expression between input and output of structures through numerical models. In this paper, we propose

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technique that combines finite element analysis (FEA), and probabilistic transformation method (PTM) to evaluate the probability density function (PDF) of response, where the expression between input and output of structures is implicit. This technique is based on the numerical simulations of the finite element analysis (FEA) and the probabilistic transformation method (PTM) by making an interface between finite element software and Matlab. Some problems of structures are treated in order to demonstrate the applicability of the proposed technique.

1. Introduction

The lack of penetration of rational uncertainty analysis methods in engineering practice is hardly due to insufficient theoretical foundations or the scarcity of efficient algorithms. Indeed, uncertainty and reliability analysis in structural engineering have been vibrant topics of research for several decades ([7], [9], [16]). While, a large portion of the associated efforts has focussed on shedding light on the fundamental and theoretical aspects, and on the application of the uncertainty analysis methods to strongly simplified, reduced-order models of structures, significant progress has also been made recently in the rational treatment of uncertainties in large FE models of complex structures.

The structural designer must verify, within a prescribed safety level, the serviceability and ultimate conditions, commonly expressed by this inequality

$$S_d < R_d,$$

where S_d represents the action effect and R_d the resistance. The intrinsic random nature of material properties and actions is actually considered by the Eurocodes ([2], [13]), which classify the methods available to deal with this randomness in three levels:

● Semi-probabilistic or level 1 methods, the most used in common practice, where the probability of failure is indirectly considered through the definition of characteristic values and the application of partial safety indexes.

● Approximate probabilistic or level 2 methods such as the first order or second order reliability methods (FORM/SORM), where the probability of failure is based on the reliability index β [10].

● Exact probabilistic or level 3 methods, where the probability of failure is computed from the joint probability distribution of the random variables associated with the actions and resistances.

A fundamental problem in structural reliability analysis is the computation of the probability integral:

$$P_f = \text{Prob}[G(X) \leq (0)] = \int_{G(X)\leq 0} f(X)\,dX, \tag{1}$$

where $X = [X_1, \cdots, X_n]^T$ in which T is the transpose, is a vector of random variables representing the uncertain parameters of considered structure, $f(X)$ is the probability density function of X, $G(X)$ is the limit state function defined such that: $G(X) \leq 0,$ is the domain of integration denoted the failure set, and P_f is the probability of failure. The difficulty of computing this integration led to development of various methods of reliability analysis such as Monte Carlo, FORM and SORM ([4], [10]), and probabilistic transformation method (PTM) [6].

Considering the properties of the structural model realistically, it is necessary to take into account some uncertainty. This uncertainty can be conveniently described in terms of probability measures, such as distribution functions. It is a major goal of reliability methods to relate the uncertainties of the input variables to the uncertainty of the structural performance. Based on their meaning in reliability methods, the sources of uncertainty may be the actions (e.g., loads, stress), or system data (geometry, boundary conditions, mass density).

The probabilistic transformation method (PTM) is an efficient reliability method to solve problems of mechanical systems with uncertain parameters. The advantage of this method is finding the "exact" expression of the probability density function (PDF) of the solution, when the PDF of the input variable is known. In many cases, the structural load effect cannot be expressed explicitly and some finite element calculations are necessary. Coupling the finite element analysis

(FEA) with the probabilistic transformation method (PTM) is therefore necessary. In this paper, a proposed method: finite element analysis (FEA) coupled with the probabilistic transformation method (PTM) is applied in order to evaluate numerically, the probabilistic and statistical characteristics of the response of stochastic mechanical system. It involves in four main steps: (1) sampling on input random variables, (2) using finite element analysis (FEA) to have the response variable of system, (3) estimating the probabilistic density function (PDF) of the response variable by using the probabilistic transformation method (PTM), (4) concluding the probability of failure and reliability of systems. To show the advantage of the proposed method, we have carried out different applications to cover several structural problems.

Notation

PTM: Probabilistic transformation method.

FE: Finite element.

PDF: Probability density function.

FEA: Finite element analysis.

FEACPTM: Finite element analysis combined to probabilistic transformation method.

FORM: First order reliability method.

SORM: Second order reliability method.

$G(.)$: Limit state function.

P_f: Probability of failure.

N_f: The number of simulation samples in which $G(.) < 0$,

N: The total number of simulation samples.

CDF: Cumulative density function.

2. Method of Analysis

2.1. General remarks

The reliability analysis of the structure with uncertain parameters requires two modelling steps, namely, the construction of a mathematical-mechanical model and the modelling of the uncertainties. For the mathematical-mechanical model of structures, the finite element analysis (FEA) is the standard tool for structure analysis, and for the second model, the concepts and notions of probability theory have been used early for capturing uncertainties in computational mechanics. In the present paper, we are interesting on evaluating the reliability of the structures with uncertain parameters based on the combination of the FEA software with the reliability method program.

2.2. Modelling of uncertainties in structural properties and loading

A most parameters which characterize any numerical model utilized in reliability analysis of structures, is affected by uncertainty ([2], [10]). Examples of such parameters are the mass density, the section dimensions of the structure or the magnitude of a load assumed to act on the considered structure. A convenient and rational way to represent this uncertainty consists in modelling, these parameters as random variables. Formally, the latter is collected in a vector of random variables.

$$X = X(\theta) = [X_1, X_2, \dots, X_d], \quad X_i = X_i(\theta), \quad \theta \in \zeta, \tag{2}$$

where θ denotes the random event, ζ the so-called *sample space*, and d is the dimension of the random vector X. The latter is characterized by its joint cumulative distribution function (CDF),

$$F_X(X) = P[X_1 \le x_1, X_2 \le x_2, \dots, X_d \le x_d], \tag{3}$$

where $X = [x_1, x_2, \dots, x_d]$ is an arbitrary value of the vector X and $P[.]$ denotes the probability of the event enclosed in the brackets. In many practical applications, numerous uncertain parameters can be assumed to be statistically mutually independent, thus, simplifying the definition of the joint CDF in (3).

2.2.1. Finite element analysis

In solving partial differential equations, the primary goal is to create an equation that approximates the equation to be studied, but is numerically stable, meaning that errors in the input data, and intermediate calculations do not accumulate and cause the resulting output to be meaningless. There are many ways of doing this, all with advantages and disadvantages. The finite element analysis is a good choice for solving partial differential equations over complex domains like mechanical system, when the domain changes (as during a solid state reaction with a moving boundary), when the desired precision varies over the entire domain, or when the solution lacks smoothness.

The finite element method is the standard tool for certain classes of partial differential equations arising in various fields of engineering and in particular for those arising in solid mechanics. For linear systems, enforcing global static or dynamic equilibrium the finite element method leads to a system of linear equations, respectively,

$$KU = F, \qquad M\ddot{U}(t) + C\dot{U}(t) + KU(t) = F(t), \tag{4}$$

where the matrices K and M are the global stiffness and mass matrices, respectively, obtained by adding the contributions of all element matrices. There are symmetrical and positive vectors.

$$K = \sum_{e} K^{e}, \tag{5}$$

$$M = \sum_{e} M^{e}. \tag{6}$$

The latter matrices have the form,

$$K^{e} = \int_{\Omega^{e}} D^{e} B^{e} d\Omega^{e}, \tag{7}$$

$$M^{e} = \int_{\Omega^{e}} \rho^{e} H^{eT} H^{e} d\Omega^{e}, \tag{8}$$

where B^e is the matrix relating element displacements and strains, D^e is the elasticity matrix relating stresses and strains, ρ^e is the mass density, H^e is the shape functions, and Ω^e is the spatial domain of the element. The global damping matrix C is typically formulated in terms of M and K.

2.3. Structural reliability analysis

2.3.1. Monte Carlo simulation

The Monte Carlo method (Rubenstein, 1981; Devictor, 1996) is used to build the PDF of the response of system, or to evaluate his probability of failure. This method involves random sampling from the distribution of input, and successive model runs until a statistically significant distribution of output is obtained [17]. This simulation techniques can be used to estimate the probability of failure defined in (1). The Monte Carlo simulation consists of drawing samples of the basic variables according to their probabilistic characteristics, and then feeding them into the performance function.

$$P_f = \frac{1}{N}\sum_{i=1}^{n} I(X_1, X_2, \dots, X_n), \tag{9}$$

where $I(X_1, X_2, \dots, X_n)$ is a function defined by the Monte Carlo method allows the determination of an estimate of the probability of failure, given by

$$I(X_1, X_2, \dots, X_n) = 1, \quad \text{if } G(X_1, X_2, \dots, X_n) \le 0, \tag{10}$$

$$I(X_1, X_2, \dots, X_n) = 0, \quad \text{if } G(X_1, X_2, \dots, X_n) > 0. \tag{11}$$

An estimate of the probability of failure P_f ([8], [11]) can be found by

$$P_f = \frac{N_f}{N}, \tag{12}$$

where N_f is the number of simulation samples, in which $G(.) < 0$, and N is the total number of simulation samples. As N approaches infinity, P_f approaches the true probability of failure. The main advantage of the Monte-Carlo method is that, this method is not only valid for static, but also for dynamic models and for probabilistic model with continuous or discrete variables. The main drawback of this method is that, it involves a long and onerous computer time.

3. Proposed Method

This paper deals with the case of coupling finite element analysis (FEA) software, and the probabilistic transformation method (PTM) program to evaluate the probability density function (PDF) of response, where the expression between input and output of the considered structure is implicit. This technique is based on the numerical simulations of the finite element analysis (FEA) and the probabilistic transformation method (PTM) by making an interface between finite element analysis software and probabilistic transformation method program. Some problems of structures are treated in order to demonstrate the applicability of the proposed technique.

3.1. Probabilistic transformation method (PTM)

The probabilistic transformation method is based on the following theorem:

Theorem. *Suppose that X is a continuous random variable with PDF (probability density function)* $f(x)$ *and* $A \subset \mathbb{R}$ *is the one-dimensional space, where* $f(x) > 0,$ *is differentiable and monotonic. Consider the random variable* $Y = u(X),$ *where* $y = u(x)$ *defines a one-to-one transformation that maps the set A onto a set* $B \subset \mathbb{R}$ *so that, the equation* $y = u(x)$ *can be uniquely solved for x in terms of y, say* $x = u^{-1}(y).$ *Then, the PDF of Y is*

$$f_Y(Y) = f_X[u^{-1}(y)]|J|, \tag{13}$$

where $J = \frac{dx}{dy} = \frac{du^{-1}(y)}{dy}$ *is the transformation Jacobean, which must be continuous for all points* $y \in B$.

The PTM is based on one-to-one mapping between the random output (s) and input (s), where the transformation Jacobean J can be computed. The PDF of the output (s) is then computed through the known joint PDF of the inputs multiplied by the determinant of transformation Jacobean matrix. The idea of PTM is based on the following formula [5]:

$$f_u(u) = |J|.f_z(z), \tag{14}$$

$$|J| = |\frac{\partial z}{\partial u}|, \tag{15}$$

where $f_u(u)$ is the probability density function of the variable u and $f_z(z)$ is the probability density function of the variable z.

The general steps in the application of the probabilistic transformation method (PTM): (1) The random variable input is generating and the stochastic equation of equilibrium is solved first by using finite element analysis software. (2) This solution is used to compute numerically the function between the input and the output, which is then inverted for the calculation of the determinant of the transformation Jacobean. Finally, the PDF of the response at any point in the domain can be deduced by using the formula (14). This is simply defined by multiplying the input PDF by the Jacobean of the inverse mechanical function. This approach has the advantage of giving a closed-form of the density function of the response, which is very helpful for reliability analysis of mechanical systems ([5], [12]).

The probabilistic transformation method (PTM) is one of the most widely used methods in reliability analysis. However, this method has drawbacks in the solution of reliability problems. It requires the evaluation of the explicit response functions with respect to the random variables, that is very difficult in analysis of complicated structures. To overcome these drawbacks, an interface between the finite element analysis and the probabilistic transformation method (PTM) is proposed in this paper.

4. Finite Element Analysis Coupled with Probabilistic Transformation Method: FEACPTM

The finite element analysis (FEA) software, is used to perform the structural analysis to obtain the structure weight, maximal displacement, and maximal stress, corresponding to a set of given design variables. These analysis results are sent to the reliability program to conduct the probabilistic density function (PDF), and the probability of failure and generate new random variables. The newly generated variables are then used to update the input file. The (FEA) software is then invoked again to perform the structural analysis with the new input parameters. This process is repeated until satisfactory results are obtained.

4.1. Finite element software

There exists large number FE software codes, but they are different implementations of the same numerical modelling and analysis methodology. In particular, most FE software codes involve three main phases, namely, (i) the pre-processing, (ii) the assembly and solution, and (iii) the post-processing.

The entire set of definitions is usually gathered into one or more input files for the finite element program, which can be used to execute the analysis in batch mode. As far as the pre-processor concerns, the finite element model is usually saved in a database file. The FE solver uses the specifications of the pre-processing phase to assemble the element matrices corresponding to the adopted formulation, e.g., the mass, damping, and stiffness matrices. The final step of the FE analysis, post-processing, it recovers the derived quantities of interest from the solution vector, which in most FE codes corresponds to the vector of the displacements at the nodal DOFs. It involves the visualization of results to facilitate their interpretation by the analyst.

4.2. Interface between FEA software and reliability analysis program

A fundamental characteristic of a software code for reliability analysis of structural engineering applications consists in the way, it interfaces with the software that gives the finite element modelling and solution. In this type of implementation, the FEA is viewed as a black box in the analysis process and the FE code is communicated with a generic interface, through the input files of the latter. The reliability analysis program controls the FE code by automatically modifying the input files, set identifiers, which govern the automatic generation of input file samples by the stochastic solver, using pattern matching and replacement.

4.3. Coupled method

By the direct coupling method, we mean any reliability procedure based on a probability of failure search algorithm using directly the FEA software each time the output of system has to be evaluated. For the application of PTM algorithm, we need to compute the function between the input and output variables of system and the determinant of Jacobean of variables, in the case, where the function between input and output variables is implicit, then we applied the spline interpolation to approximate this function, and calculating numerically the Jacobean of the input and output variables. These steps are discussed in previous subsections.

4.3.1. Spline interpolation

In the mathematical field of numerical analysis, spline interpolation is a form of interpolation, where the interpolate is a special type of piecewise polynomial, called a *spline*. Spline interpolation is preferred over polynomial interpolation because the interpolation error can be made small even, when using low degree polynomials for the spline. Thus, spline interpolation avoids the problem of Runge's phenomenon, which occurs when using high degree polynomials.

Given $n + 1$ distinct knots x_i such that

$$x_0 < x_1 < \ldots < x_{n-1} < x_n. \tag{16}$$

For a data set x_i of $n + 1$ points, we can construct a cubic spline with n piecewise cubic polynomials between the data points. If

$$f(x) = \begin{cases} f_0(x) & x \in [x_0, x_1], \\ f_1(x) & x \in [x_1, x_2], \\ \vdots & \vdots \\ f_n(x) & x \in [x_{n-1}, x_n], \end{cases} \tag{17}$$

represents the spline function interpolating the function f, we require the interpolating property,

$$f(x_i) = S(x_i), \tag{18}$$

the splines to join up are

$$f_{i-1}(x_i) = f_i(x_i),\ i = 1, \ldots, n-1, \tag{19}$$

the function f twice continuous differentiable,

$$f'_{i-1}(x_i) = f'_i(x_i) \text{ and } f''_{i-1}(x_i) = f''_i(x_i),\ i = 1, \ldots, n-1. \tag{20}$$

For the n cubic polynomials comprising S, this means to determine these polynomials, we need to determine $4n$ conditions (since for one polynomial of degree three, there are four conditions on choosing the curve). However, the interpolating property gives us $n+1$ conditions, and the conditions on the interior data points give us $n+1-2 = n-1$ data points each, summing to $4n-2$ conditions. We require two other conditions, and these can be imposed upon the problem for different reasons.

One such choice results in the so-called clamped cubic spline, with

$$f'(x_0) = u, \tag{21}$$

$$f'(x_k) = v, \tag{22}$$

for given values u and v.

Alternately, we can set

$$f''(x_0) = f''(x_n) = 0. \tag{23}$$

Interpolation using natural cubic spline can be defined as

$$f_i(x) = \frac{Z_{i+1}(x - x_i) + Z_i(x_{i+1} - x)}{6h_i} + \left(\frac{y_{i+1}}{h_i} - \frac{h_i z_{i+1}}{6}\right)(x - x_i), \tag{24}$$

and

$$h_i = x_{i+1} - x_i. \tag{25}$$

The coefficients can be found by solving this system of equations

$$Z_0 = 0, \tag{26}$$

$$h_{i-1}Z_{i-1} + 2(h_{i-1} + h_i)Z_i + h_i Z_{i+1} = 6\left(\frac{y_{i+1} - y_i}{h_i} - \frac{y_i - y_{i-1}}{h_{i-1}}\right),$$

$$i = 1, \cdots, n - 1. \tag{27}$$

In order to interpolate the function between the input variable and the output variable, we are choosing a *B*-spline curve, this schema converge faster towards the target curve and produces a better approximation curve than existing methods relying on data point parameterization. In order to applique the PTM, we need to calculate the determinant of Jacobean of input and output variables.

4.3.2. Calculation of the determinant of Jacobean of variables

In our case, we have the determinant of Jacobean of one input and one output variable:

$$|J| = \left|\frac{\partial z}{\partial u}\right|. \quad \text{(From 15)}$$

The input variable is generating, and for each input variable, the output variable is calculating by using FEA software and communicated to reliability program for approximating the function between this variables by spline interpolation. When we have the interpolate function, numerical schema used for computing the determinant of the Jacobean.

4.4. Algorithm of method FEACPTM

The outline of a proposed method FEACPTM is as follows:

(1) Generate the input random variables;

(2) Calculate the value of output variables by FE software, for each value of input the correspondent value of output is estimating by using FEA and stocking it in solution file;

(3) Approximate the function between input and output variables by using spline interpolation;

(4) Calculate the determinant of Jacobean of input and output variables; (by 15)

(5) Applique the basic relation of PTM; (by 14)

(6) Evaluate the graphic of PDF of output variable in function of this output variables (in our case, the PDF of displacement in function of displacement);

(7) Approximate the probability of failure P_f.

The algorithm of proposed technique illustrated by this Figure 2.

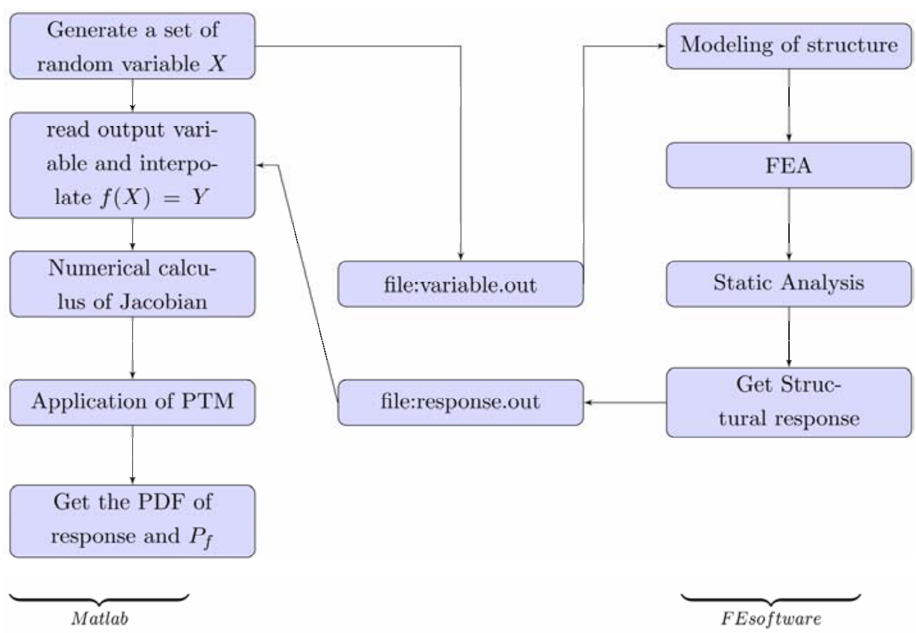

Figure 1. Algorithm of FEACPTM.

5. Application

The examples presented in this section have implicit function of output variable in terms of the input random variable. The proposed method uses the combination with computationally demanding procedures involving the FEA for modelling the structure in every iteration, and calculate the value of displacement (output variable) corresponding at each input variable value, in other hand, it would be necessary to use the PTM program for estimating the value of P_f of the considered structure.

5.1. 3 bars

In the first application, we are going to analyze the reliability of a structure constituted by 3 bars (Figure 2) with random parameters (Young's modulus *E*). Geometrical and material properties are:

The section of each beam $A = 1\text{m}^2$.

The length $l = 3\text{m}$.

The load $W = 1, 2\text{N}$.

$\rho = 7800\text{kg} / \text{m}^3$.

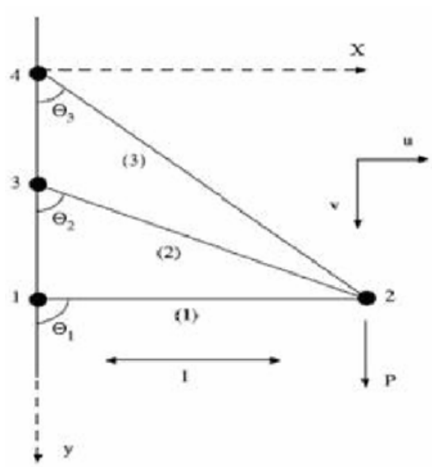

Figure 2. The 3 bars.

E is uniformly distributed in the range [10, 11]. Using the proposed technique FEACPTM, we obtain the following graph:

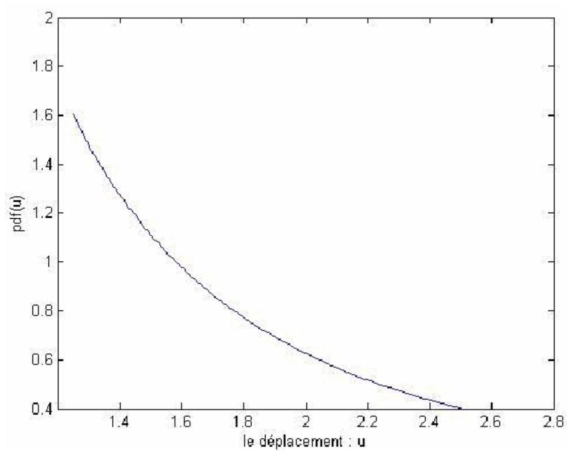

Figure 3. PDF (u) when E is uniformly distributed.

The PDFs of the normalized vertical displacement u_y are plotted in Figure 3 assuming that the variable u_y are independent and uniformly distributed in the range [1.2505; 2.5009]. Also in this case, the results are accurate as shown by favorable comparison with classical Monte Carlo simulation. Let us suppose the limit displacement is $u_{\text{limit}} = 2\text{mm}$. It is required to find the failure probability $P_f = P(u \leq u_{\text{limit}})$.

The numerical values of probabilistic characteristics of the displacement of this truss is listed in this table.

Table 1. Results obtained by FEACPTM and Monte Carlo simulation

	FEACPTM	Monte Carlo simulation	MSE	APE (%)
U_{min}	1.2505	1.2505	0	0
U_{max}	2.5009	2.5008	$1.0e-08$	0.004
U_{mean}	1.7349	1.7334	$2.250e-06$	0.0865
var	0.3556	0.3496	$3.60e-05$	1.7162
P_f	0.2536	0.2504	$1.0240e-05$	1.278

The evaluated failure probability and the values of displacement show that, the results are the same with the FEACPTM and the Monte Carlo simulation. It is therefore proved by comparison that the results are validated. In other hand, the values of MSE do not exceed $1.0240e - 05$, and the values of APE do not exceed 1.7162%. It can be clearly seen the performance of our technique proposed.

5.2. Truss

This application treats structural analysis on one hand of the arrow of a crane of construction that one assimilates to a spatial truss. It is constituted by bars that are identical. To the extremity of this truss is applied a load $M = 5t$. The bars forming the structure are in steel of which Young's modulus E = [100GPa, 300GPa] and the Poisson coefficient $\sigma = 0.29$.

The weight of each bar of the truss is not negligible in front of the load *M*. The goal of this analysis is to determine the efforts, the constraints in the different elements of the truss, and the maximum displacement generated by the applied load to his extremity.

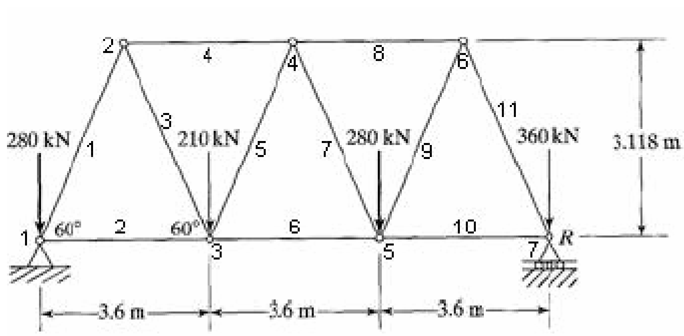

Figure 4. The truss formed by 11 beams.

We are study the truss with ANSYS.

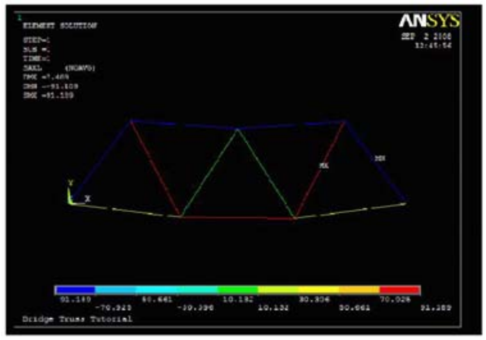

Figure 5. The deformation of truss.

E is uniformly distributed in the range E = [100GPa, 300GPa] with E_{moy} = 200GPa. Using the proposed technique FEACPTM, we obtain the following graph.

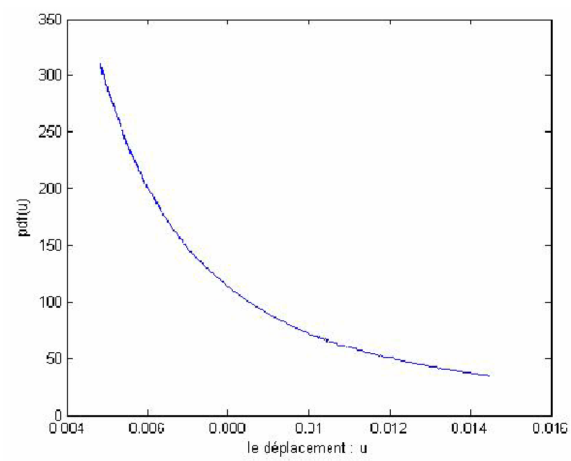

Figure 6. The truss formed by 11 beams.

The PDFs of the normalized vertical displacement u_y are plotted in Figure 6 assuming that the variable u_y are independent and uniformly distributed in the range [$4.80e-03$; $1.450e-02$]. Also in this case, the results are accurate as shown by favorable comparison with classical

Monte Carlo simulation. Let us suppose the limit displacement is $u_{\text{limit}} = 1.23e - 02\text{mm}$. It is required to find the failure probability $P_f = P(u \leq u_{\text{limit}})$.

The numerical values of probabilistic characteristics of the displacement of this truss is listed in this table.

From the Table 2, the results demonstrate that the technique FEACPTM for reliability analysis allows us to obtain the results with small MSE (not exceed $2.2658e-005$) and APE (not exceed 5.3883%) compared to results obtained by Monte Carlo simulation. It is of great importance for the reliability analysis of complex structures.

Table 2. Results obtained by FEACPTM and Monte Carlo simulation

	FEACPTM	Monte Carlo simulation	MSE	APE (%)
$U_{\min}$	$4.80e - 03$	$4.824e - 03$	$5.760e - 10$	0.4975
$U_{\max}$	$1.450e - 02$	$1.447e - 02$	$9.0e - 10$	0.2073
U_{mean}	$8e - 03$	$7.949e - 03$	$2.601e - 09$	0.6416
var	$2.6e - 03$	$2.572e - 03$	$7.8400e - 010$	1.0886
P_f	$9.31e - 02$	$8.834e - 02$	$2.2658e - 05$	5.3883

5.3. Industrial application

We are going to analyze the reliability of the pylon of a line of transportation of electricity that one assimilates to a truss plan. Two identical loads F of 1.8KN are applied to the two superior extremities of the following pylon an angle of $\theta = 15°$. The bars forming the pylon are in steel of which Young's modulus E = [100GPa, 300GPa] and the Poisson coefficient $\sigma = 0.29$. The section of every bar is worth $A = 27.90\text{cm}^2$. The hypothesis for this problem is that, the weight of each bar of the pylon is negligible in front of the applied efforts.

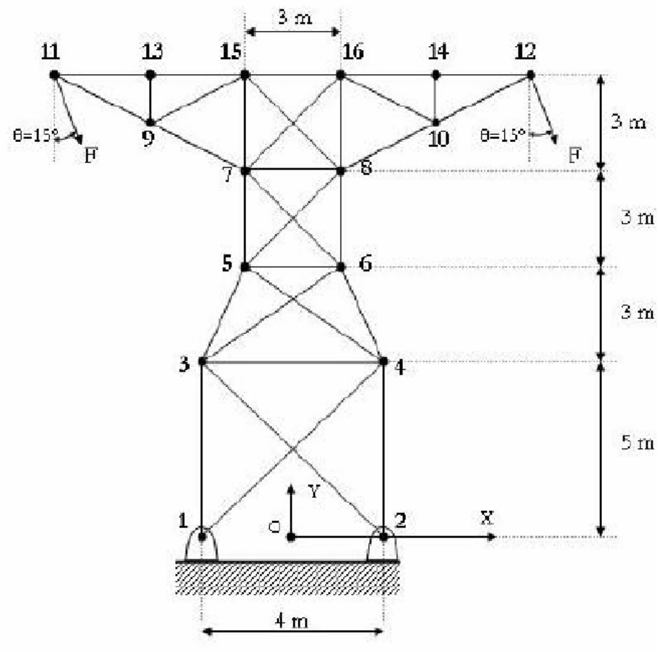

Figure 7. Pylon.

The Young's modulus E is uniformly distributed in the range E = [100GPa, 300GPa]. Using the proposed technique FEACPTM, we obtain the following graph:

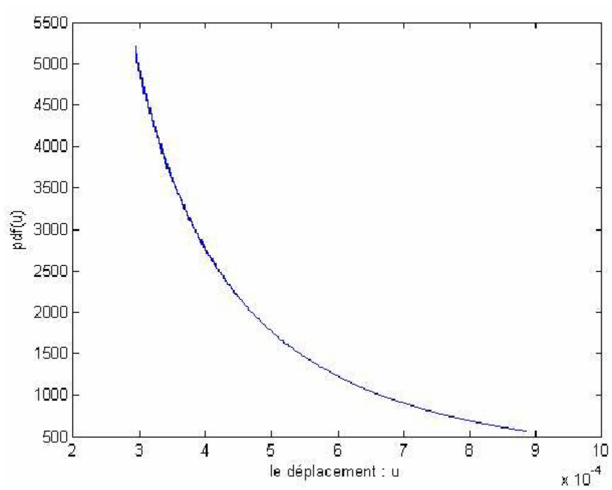

Figure 8. PDF (u) when E is uniformly distributed.

The PDFs of the normalized vertical displacement u_y are plotted in Figure 8 assuming that the variable u_y are independent and uniformly distributed in the range [2.9489e – 04; 8.8466e – 04]. Also in this case, the results are accurate as shown by favorable comparison with classical Monte Carlo simulation. Let us suppose the limit displacement is $u_{\text{limit}} = 6.635e - 04\text{mm}$. It is required to find the failure probability $P_f = P(u \leq u_{\text{limit}})$.

The numerical values of probabilistic characteristics of the displacement of this pylon is listed in this table.

Table 3. Results obtained by FEACPTM and Monte Carlo simulation

	FEACPTM	Monte Carlo simulation	MSE	APE(%)
$U_{\min}$	$2.9489e-04$	$2.9490284e-04$	$1.6487e-016$	0.0044
$U_{\max}$	$8.8466e-04$	$8.8460969e-04$	$2.5311e-015$	0.0057
U_{mean}	$4.8605e-04$	$4.8594847e-04$	$1.0308e-014$	0.0209
var	$1.5756e-04$	$1.5725825e-04$	$9.1053e-014$	0.1919
P_f	$1.8856e-01$	$1.66693e-01$	$4.7817e-04$	13.1181

Table 3 reports the results obtained by our technique and the Monte Carlo simulation (10000 iterations). This table also illustrates the efficiency of the FEACPTM, since a number of 800 (less than 1000) iterations suffices to obtain results close to those obtained by Monte Carlo simulation. To compare the results, the MSE and APE are calculated. The values of MSE and APE are very small, which shows the accuracy and efficiency of FEACPTM.

6. Conclusion

In this paper, we are analyzing the structures with parameter uncertainties. The uncertainty has been considered in the material properties as well as in the Young's modulus, load, section, etc. An efficient, accurate, robust algorithm is proposed to solve the reliability problem with implicit response functions. The proposed algorithm integrates the treatment by the finite element analysis method with FEA software and the probabilistic transformation method (PTM) (reliability program). In the proposed method, the finite element analysis is used to approximate the structural response function. Once the implicit response function is found numerically, the probabilistic transformation method (PTM) can be easily applied to solve the complicates structural reliability problem. The accuracy and efficiency of the proposed method is demonstrated through numerical examples of structures.

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